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Banking and the advantage of hedging

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Abstract

In this paper, we study how a competitive banking firm can use a variable deposit rate to insure against profit risk from risky assets and how the utility of the bank manager is affected by this kind of risk management policy. Furthermore, we study the advantage of a risk management policy which is based on financial hedging. Finally, we answer the question which of these risk management policies the bank manager prefers.

Statement of relationship to practice

The ultimate goal of risk management in banking firm is to facilitate a consistent implementation of risks and asset/liability policy. Traditional risk practices consist of setting risk limits. Modern best practices consist of setting riks limits based on economic measures of risk while ensuring the best risk-adjusted performances. The goal of the banking firm is to enhance the risk-return profile of transactions and the bank's portfolio. We analyze the advantage of risk management which is based on financial hedging.

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1. Introduction

An increase in the volatility of interest rates and the ongoing globalization and deregulations of banks and capital markets have induced financial intermediaries to be prominent participants in risk sharing markets. A banking firm has to decide on how much to invest in risky assets, how much to attract deposits and to what extent to transfer risk to their customers given the current capital position. The use of variable rate of deposits is one possibility of risk transfer. When there are financial hedging possibilities available the bank may use financial derivatives to deal with the bank's exposure. In a partial equilibrium analysis, we compare and evaluate these two risk management policies in the context of a widely used model of the banking firm (Klein (1971), Santomero (1984), Morgan et al (1988), Chang, Rhee and Wong (1995), Freixas and Rochet (1997), Wahl and Broll (2005), Broll and Eckwert (2006)). We are interested in answering the question which of the two risk management techniques the risk-averse bank manager prefers.

The paper proceeds as follows. Section 2 presents the formulation of a model of a competitive banking firm under uncertainty. We investigate the impact of expected marginal income, return risk, and equity on the volume of deposits and investment. In section 3 we discuss two fundamental ways of risk transfer: rolling over risk to the customers of the bank or selling risk in risk sharing markets. We demonstrate that both alternatives have different effects on profit risk. Therefore in section 4 we show that risk management by financial hedging is preferred to transfering risk by firm-specific arrangements. A final section concludes the paper.

2. The model and basic results

Assume the bank has a one-period planning horizon. The bank's investment in risky assets, A, is financed by deposits (net of reserve requirements), D, and fixed equity $\overline{K} > 0$. The returns on the assets, $\tilde{r}$, are stochastic. At the beginning of the period, the bank manager must decide on the amount of deposits, subject to the bank's balance sheet constraint, $A = \overline{K} + D$ (see, for example, Wong (1997), Broll and Guinnane (1999)).

The cost of funds of the bank, $C(D)$, are known and increasing, i.e. $C'(D) > 0$. Furthermore $C''(D) > 0$. We assume that $C'(0) = 0$ and $C'(D) \to \infty$ as $D \to \infty$. Hence the optimum amount of deposits is positive under certainty. Additionally, the bank bears fixed setup cost F. The bank manager is risk-averse and possesses a von Neumann-Morgenstern utility function, U, with properties $U' > 0$ and $U'' < 0$. For simplicity, we assume throughout the paper that all institutional solvency requirements are satisfied (for a discussion of bank regulation see, for example, Dewatripont and Tirole (1994), Greenbaum and Thakor (1995), Wong (1997)).

Financial intermediation. Let us consider a price-taking bank which faces a given rate of deposits, r_D. Then the bank's random profit, $\tilde{\Pi}$, can be stated as

$$\tilde{\Pi} = \tilde{r}A - r_D D - C(D) - F. \tag{1}$$

Note that profit is equivalent to incremental end-of-period wealth, if variable and the fixed cost of the bank are compounded to the end of the period. At the beginning of the period, that is to say, before the return on risky assets is revealed, the bank manager has to choose the amount of deposits to maximize expected utility of profit:

$$\max_{D \geq 0} E[U(\tilde{\Pi})],$$

where $\tilde{\Pi}$ is defined in (1), subject to the bank's balance sheet constraint

$$A = \overline{K} + D. \tag{2}$$

The first-order condition for an optimum decision reads

$$E[U'(\tilde{\Pi}^*)(\tilde{r} - r_D - C'(D^*))] \leq 0, \tag{3}$$

where C' denotes marginal cost, U' is marginal utility of profit, and the asterisk denotes optimal levels.

The bank acts as a financial intermediary only if the bank margin, i.e. the expected marginal income, is positive. This follows from the fact that, if

$$E[U'(\tilde{\Pi}^*)(\tilde{r} - r_D - C'(D^*))] < 0, \tag{4}$$

then $D^* = 0$ (see Arrow (1965)). Condition (4) implies $E(\tilde{r}) - (r_D + C'(D^*)) \leq 0$, where this term represents the optimal bank margin. Thus the bank manager limits the investment in risky assets to the amount of fixed equity if the bank margin is negative (or, zero). In this case the bank cannot meet its function as a financial intermediary.

Certainty (equivalent) case. Assume that the bank's optimum amount of deposits is positive. Let us consider the relationship between optimal levels of deposits under uncertainty and under its certainty equivalent, respectively.

Since profit Π^* increases when r increases and since marginal utility is a decreasing function of profit, we get $\text{cov}(U'(\tilde{\Pi}^*), \tilde{r}) < 0$. Therefore, equation (3) implies that the bank margin is positive:

$$E(\tilde{r}) - (r_D + C'(D^*)) > 0. \tag{5}$$

Denote by D_c^* the optimal amount of deposits when $r_c = E(\tilde{r})$ is the certain return on risky assets, i.e. the so-called certainty (equivalent) case (Leland (1972)). Then from the optimality condition for the certainty case, we get equality between marginal revenue and marginal cost, i.e.

$$r_c = r_D + C'(D_c^*). \tag{6}$$

Since marginal cost is strictly increasing, condition (5) and equation (6) imply that the optimum amount of deposits reduces when uncertainty occurs, i.e. $D^* < D_c^*$. The reason is that the bank manager is risk-averse and that the bank has to bear all the risk.

Increase in risk. The effect of risk, shown in the certainty (equivalent) case, cannot be generalized in the sense that the optimal amount of deposits is monotonically decreasing in risk. The lack of monotonicity can be avoided by specific assumptions about the utility function.

Following the literature one can study the effect of an increase in the risk of the rate of return on the assets by replacing $\tilde{r}$ with $\tilde{r}'$, where $\tilde{r}'$ is a mean-preserving spread (MPS) of the risky rate of return $\tilde{r}$. We make the following assumption about preferences.

Assumption (*A.1*): Let $U'(x)$ be strictly convex and $xU'(x)$ be concave.

Assumption (A.1) holds, for example, when (1) U is a logarithmic utility function, or a generalized logarithmic utility function of the form $U(\Pi) = \Pi + \gamma \log(\Pi), \gamma > 0$, or (2) U is a (negative) exponential utility function, or a one-switch utility function $U(\Pi) = \Pi - \gamma \exp(-2\Pi), \gamma > 0$ (Pratt and Zeckhauser (1987)), if $\Pi \leq 1$. Note that the preferences of assumption (A.1) imply that relative prudence is less or equal 2. Under assumption (A.1) the impact of a mean-preserving increase in return risk of the assets is found by using the first-order condition (3). Since $U'(x)$ is strictly convex while $xU'(x)$ is concave we have

$$E[U'(\tilde{\Pi}'); \tilde{r}'] \quad > \quad E[U'(\tilde{\Pi}); \tilde{r}], \tag{7}$$

$$E[\tilde{r}'U'(\tilde{\Pi}'); \tilde{r}'] \quad \leq \quad E[\tilde{r}U'(\tilde{\Pi}); \tilde{r}]. \tag{8}$$

Multiplying inequality (7) by $r_D + C'(D^*)$ and subtracting the result from condition (8) we obtain from the first-order condition (3), where the optimal amount of deposits D^* remains unchanged, the following inequality:

$$E[U'(\tilde{r}'(\overline{K} + D^*) - r_D D^* - C(D^*) - F)(\tilde{r}' - r_D - C'(D^*))] < 0. \tag{9}$$

In order to satisfy the first-order condition after the MPS in the risky return the optimum amount of deposits before the spread must be reduced. Therefore we must have $D^{*\prime} < D^*$.

Due to the balance sheet constraint, note that in the certainty (equivalent) case as well as in the MPS-case, the bank's investment in the risky assets declines as a result of uncertainty.

Increase in equity. If equity of the bank increases, although the intermediary has more financial resources to invest in risky assets and to attract more deposits, the impact of equity on the level of deposits is ambiguous, in general. Specific assumptions about the utility function can resolve the ambiguity. This can be shown by a comparative-static analysis of the first-order condition (3) for $D^* > 0$.

Assumption (*A.2*): Let $U(x)$ exhibit constant or decreasing relative risk aversion.

Assumption (A.2) holds, for example, if U represents hyperbolic absolute risk aversion preferences, i.e. $U(\Pi) = (\gamma/\theta + \Pi)^{1-\theta}/(1-\theta)$, with parameter values $\gamma \leq 0$ and $0 < \theta \neq 1$.

We have ($\tilde{M}^* \equiv \tilde{r} - r_D - C'(D^*), N^* \equiv r_D D^* + C(D^*) + F$):

$$\begin{aligned} \text{sign}\,\frac{dD^*}{d\bar{K}} &= \text{sign}\,E[U''(\tilde{\Pi}^*)\tilde{M}^*\tilde{r}] \\ &= \text{sign}\,\{E[U''(\tilde{\Pi}^*)\tilde{\Pi}^*\tilde{M}^*] + E[U''(\tilde{\Pi}^*)\tilde{M}^*]N^*\}. \end{aligned} \tag{10}$$

Using the Arrow-Pratt measure of absolute and relative risk aversion, $R_a(x) = -U''(x)/U'(x)$, respectively, $R_r(x) = -U''(x)x/U'(x)$, we get

$$E[U''(\tilde{\Pi}^*)\tilde{M}^*] = -E\{[R_a(\tilde{\Pi}^*) - R_a(\hat{\Pi})]U'(\tilde{\Pi}^*)\tilde{M}^*\}, \tag{11}$$

$$E[U''(\tilde{\Pi}^*)\tilde{\Pi}^*\tilde{M}^*] = -E\{[R_r(\tilde{\Pi}^*) - R_r(\hat{\Pi})]U'(\tilde{\Pi}^*)\tilde{M}^*\}, \tag{12}$$

where we have used equation (3), and $\hat{\Pi}$ is the profit level at which the stochastic variable $\tilde{M}^*$ equals zero. It follows that the LHS of equation (11) and, respectively, the LHS of equation (12) is positive (zero) [negative] if absolute, respectively, relative risk aversion is decreasing (constant) [increasing] (Arrow (1965)). Since nonincreasing relative risk aversion implies decreasing absolute risk aversion, we conclude from equation (10) in combination with equations (11) and (12) that $dD^*/d\overline{K} > 0$, i.e. the optimum amount of deposits increases with the endowment of equity, if relative risk aversion does not increase with profit.

Note that with nondecreasing absolute risk aversion (for example, a (negative) exponential or a quadratic utility function) the optimum amount of deposits will strictly decrease when equity increases. Furthermore, the propounded hypotheses of risk behavior by Arrow (1965), i.e. increasing relative risk aversion while absolute risk aversion is decreasing does not allow to answer to question how equity affects the optimum amount of deposits of our competitive banking firm.

Increase in fixed cost. Let us briefly consider a comparative-static analysis of equation (3) regarding the effect of fixed setup cost upon the optimum volume of deposits. We obtain

$$\operatorname{sign}\frac{dD^*}{dF} = -\operatorname{sign} E[U''(\tilde{\Pi}^*)\tilde{M}^*] = \operatorname{sign} R'_a(\Pi), \tag{13}$$

where $R'_a(\Pi)$ is the derivative of absolute risk aversion with respect to profit. Equation (13) follows immediately from our above discussion on increases in equity. Hence, the optimum volume of deposits decreases (does not change) [increases], if and only if absolute risk aversion is decreasing (constant) [increasing]. This is the well-known wealth effect on investment in risky assets (Arrow (1965)).

The discussion of the basic implications of our model is summarized in the next proposition.

Proposition 1 *(Financial intermediation)*

1. *A negative bank margin is sufficient to deter the bank manager from attracting deposits.*

2. *In the certainty (equivalent) case, i.e. when the certain return on investment in assets equals the expected return on investment in risky assets, the amount of deposits as well as the bank's investment in assets decline as a result of uncertainty.*

3. *A mean-preserving spread in the risky return on the assets implies that the bank's business activities may decline.*

4. *Higher equity of the bank to begin with does not necessarily mean higher optimum volume of deposits. However, if relative risk aversion is constant or decreasing, then the bank's optimum amount of deposits will be higher the higher the endowment in equity.*

5. *Higher fixed setup cost of the bank imply lower optimum volume of deposits if and only if absolute risk aversion is decreasing.*

Corollary *Assume nonincreasing relative risk aversion. If the bank margin is positive, then $D^* > 0$.*

Proof. Suppose $\overline{K} = 0$. Then with $E(\tilde{r}) - r_D - C(D^*) > 0$ the first-order condition can only be satisfied if $D^* > 0$. Suppose $\overline{K} > 0$. Then by Proposition 1 (4) positive D^* will further increase. □

The discussion of this section assumed that the bank has to bear all the economic risk. In the real world there exists a variety of instruments to share the risk with other economic agents.

3. Risk transfer

Let us know consider the case where the bank manager has the opportunity to transfer risk to other parties. Basically, there are two ways to allocate the bank's economic risk: first, the risk is rolled over to the customers of the bank and second, the risk is sold in a market.

3.1 Variable deposit rate

Assume that the bank manager is able to adopt a policy of rolling over the economic risk of the bank to their customers via a variable deposit rate, $\tilde{r}_D$. By contract, the variable deposit rate is related to the random rate of return on the risky assets, $\tilde{r}_D = \phi(\tilde{r})$, with $\phi' \neq 0$. To make the model more tractable we make the following simplifying assumption:

Assumption (*A.3*): The variable deposit rate is determined by the relation

$$\tilde{r}_D = \tilde{r} - \alpha, \tag{14}$$

where $\alpha > 0$ is a known mark-down.

The mark-down α guaranties that the bank has an incentive to act as a financial intermediary. Furthermore, the transfer of risk per unit of deposits to the bank's customer is complete, see equation (14), i.e. the contractual design holds for every state of nature.

The decision maker's problem is to choose the amount of deposits so as to maximize expected utility of profit, where the bank's random profit is given by

$$\tilde{\Pi} = \tilde{r}A - \tilde{r}_D D - C(D) - F, \tag{15}$$

subject to the balance sheet constraint (2) and the contractual arrangement (14) of the variable rate of deposits.

The first-order condition is

$$E[U'(\tilde{\Pi}^*)(\alpha - C'(D^*))] = 0. \tag{16}$$

Therefore the optimum volume of deposits is determined by

$$\alpha = C'(D^*). \tag{17}$$

We claim the following proposition.

PROPOSITION 2 *(Roll-over transfer of risk)*

1. *The bank's optimal amount of deposits is independent of the bank manager's attitude towards risk, her probability beliefs, the amount of equity and the fixed setup cost.*
2. *Rolling over risk by a complete contract does not imply a riskless bank profit.*

PROOF. (1) is a direct implication of equation (17). (2) In the optimum the bank's profit becomes

$$\tilde{\Pi}^* = \tilde{r}\overline{K} + \alpha D^* - C(D^*) - F. \tag{18}$$

Hence the bank's profit remains risky. □

Proposition 2 states that when the contractual design of a variable deposit rate is given according to assumption (A.3) then the optimal level of deposits of our competitive banking firm depends only upon marginal cost and marginal revenue like in the certainty case. Furthermore, the optimal level of deposits increases with marginal revenue α, beliefs and preferences have no impact. Also, the magnitudes of equity and fixed setput cost are irrelevant for optimum decision making. However, since the balance sheet constraint has to hold, rolling over risk by the variable rate of deposits will not eliminate profit uncertainty.

3.2 Financial hedging

Now consider the case when economic risk of the bank is sold in a market instead of rolled over via a variable rate of deposits. The banking firm can enter a competitive futures market, where r_f denotes the unbiased forward rate at the beginning of the period, i.e. $r_f = E(\tilde{r})$, for the delivery at the end of the period. Just as the amount of deposits, D, issued by the bank, the volume of the bank's futures contracting, H, has to be determined at the beginning of the planning period.

With futures hedging bank's profit at the end of the period is given by

$$\tilde{\Pi} = \tilde{r}A - r_D D - C(D) - F + H(r_f - \tilde{r}), \qquad (19)$$

where $H > 0$ means that the bank is selling in the futures market. For simplicity, initial margins and variation margin calls are ignored.

Before any uncertainty is resolved, the bank chooses its amout of deposits and the futures hedge so as to maximize expected utility of profit:

$$\max_{D,H} E[U(\tilde{\Pi})], \tag{20}$$

where $\tilde{\Pi}$ is defined in (19), subject to the balance sheet constraint (2).

The first-order conditions for an optimum of decision problem (20) are:

$$E[U'(\tilde{\Pi}^*)(\tilde{r} - r_D - C'(D^*))] = 0, \tag{21}$$

$$E[U'(\tilde{\Pi}^*)(r_f - \tilde{r})] = 0. \tag{22}$$

We claim the following proposition.

Proposition 3 *(Market transfer of risk)*

1. *(Separation) When financial futures markets are available, the bank's optimal decisions about the amount of deposits (cum investment) are independent of the bank manager's attitude towards risk and the distribution function of the random return. Furthermore, equity and fixed setup cost have no impact on the bank's optimal asset and deposit management.*

2. *(Futures contracting) With unbiased futures markets the bank is completely hedged.*

Proof. (1) Rearranging and substituting condition (22) into condition (21), we get

$$r_f = r_D + C'(D^*). \tag{23}$$

That is, in the optimum marginal cost equal the forward rate. We see from (23) that the optimal amount of deposits is neither dependent upon the degree of the risk aversion of the decision maker nor on the stochastic properties of the risky return. Also $\overline{K}$ and F do not occur in the decision rule (23).

Since $r_f = E(\tilde{r})$ equation (22) reduces to

$$\mathrm{cov}(U'(\tilde{\Pi}^*), \tilde{r}) = 0, \tag{24}$$

implying that optimum profit of the banking firm has to be constant. Hence, we must have $H^* = A^*$, for then

$$\Pi^* = r_f(\overline{K} + D^*) - r_D D^* - C(D^*) - F, \tag{25}$$

is indeed riskless and the covariance is zero. The optimal hedge volume must be a full hedge. □

Note that in both cases, the roll-over transfer of risk and the market transfer of risk case characteristics of the decision makers preferences besides positive and decreasing marginal utility of profit do not matter for optimum decision making. Also, the expectations of the decision maker regarding future asset returns have no importance for optimum asset management.

4. The preferred risk management policy

In the preceeding sections we have analyzed the implications of different ways to transfer risk coming out of the optimal business activity of the competitive banking firm. Now we would like to investigate the welfare implications of the roll-over transfer of risk and the market transfer of risk, respectively.

The bank manager will prefer the risk sharing alternative which provides maximum welfare.

For an unbiased futures market and a certainty (equivalent) rate of deposits, i.e. $r_D = E(\tilde{r}_D)$ we claim:

PROPOSITION 4 *(Advantage of financial hedging) The bank manager prefers risk management by financial hedging to risk management by choosing a variable rate of deposits.*

PROOF. Let D_R^* (D_H^*) denote optimum volume of deposits in the roll-over (market) transfer of risk case. (a) Rolling over risk leads to $\alpha = C'(D_R^*)$ (see equation (17)). By financial hedging we obtain $r_f - r_D = C'(D_H^*)$ (see equation (23)). With unbiased futures markets, i.e. $r_f = E(\tilde{r})$, and a certainty (equivalent) rate of deposits, i.e. $r_D = E(\tilde{r}_D)$, we have $D_R^* = D_H^*$ since the contractional design of the variable rate of deposits implies $\alpha = r_f - r_D$. (b) In the case of roll-over transfer of risk, maximum expected utility, V_R^*, is given by ($k^* \equiv \alpha D^* - C(D^*) - F; D^* = D_R^* = D_H^*$):

$$V_R^* = E[U(\tilde{r}\overline{K} + k^*)]. \tag{26}$$

Maximum expected utility with financial hedging, V_H^*, reads

$$V_H^* = U(E(\tilde{r})\overline{K} + k^*). \tag{27}$$

From risk aversion we get $V_R^* < V_H^*$ by Jensen's inequality which proves the claim. □

The intuition behind Proposition 4 is as follows. Rolling over the economic risk to the customers of the bank only imperfectly eliminates profit risk.

The bank still has to bear some risk, since as a financial intermediary the balance sheet constraint has to hold and endowment in equity is positive. Alternatively, using unbiased futures markets as a risk management device leads to a perfect hedge of the bank's exposure. The bank does not have to bear any risk.

Hence, competitive banks prefer financial hedging instruments to firm-specific risk transforming arrangements to control their exposure to investment risk from asset management.

5. Concluding remarks

Risk management is critical in enabling competitive banking firms to make valuable investment decisions (Stulz (1996)). Our paper is a contribution to the study of the advantage of financial hedging to control profit risk over a transfer of risk through the contractual design of deposits. Our study shows that under widely used conditions risk management which uses risk sharing markets provides the highest welfare to the bank manager. Furthermore, risk management can be designed in such way that neither the bank nor their customers have to bear any risk.

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